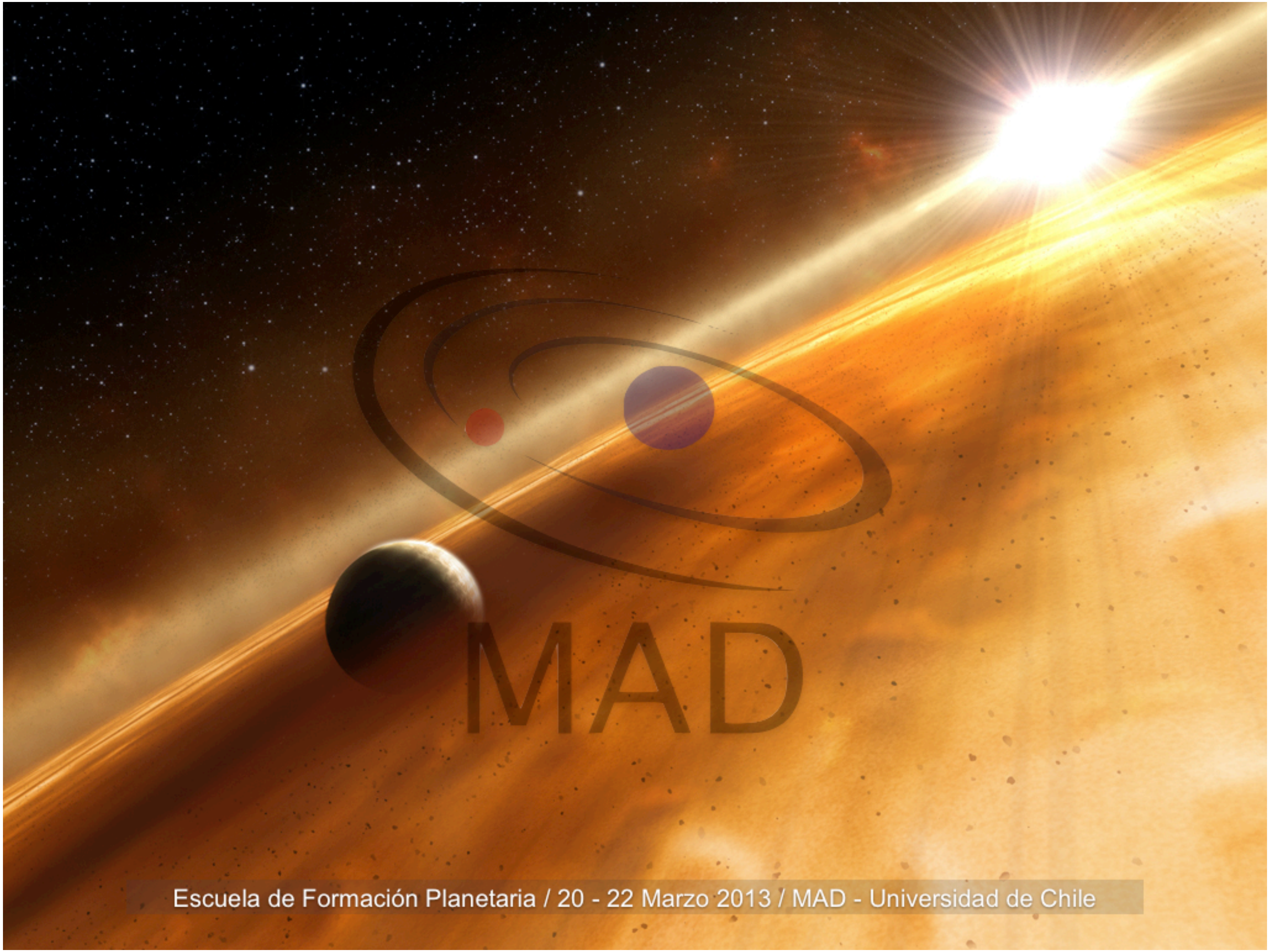
MAD
Escuela de Formación Planetaria / 20 - 22 Marzo 2013 / MAD - Universidad de Chile

DISK MODELLING . Part I

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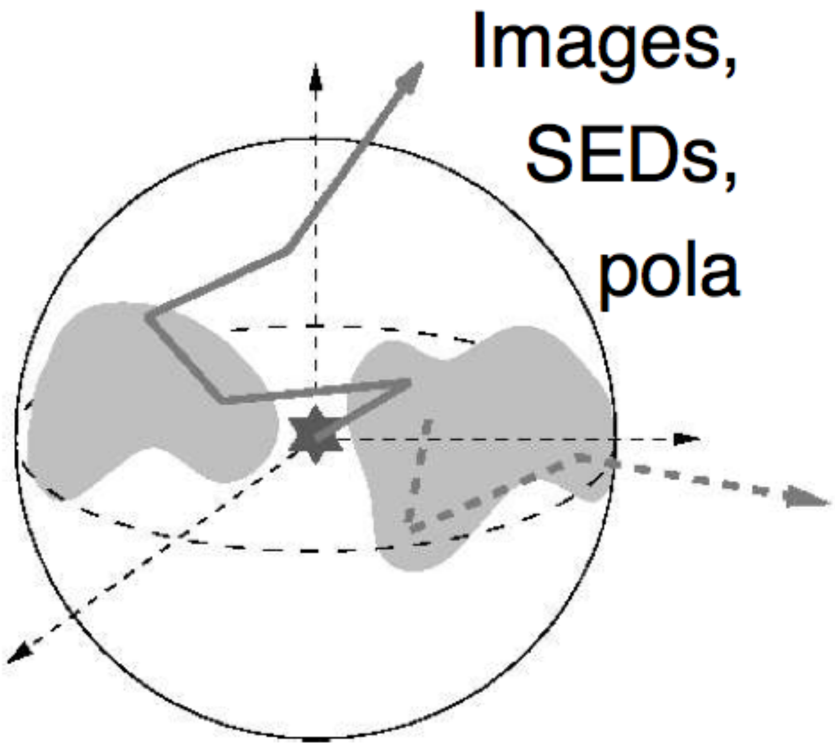
Outline

- The goals and equations of RADIATIVE TRANSFER
- Application of Radiative Transfer to light scattering and polarisation by small dust particles
- An example of how to convert Astronomical Data into disk and dust properties
- Final remarks

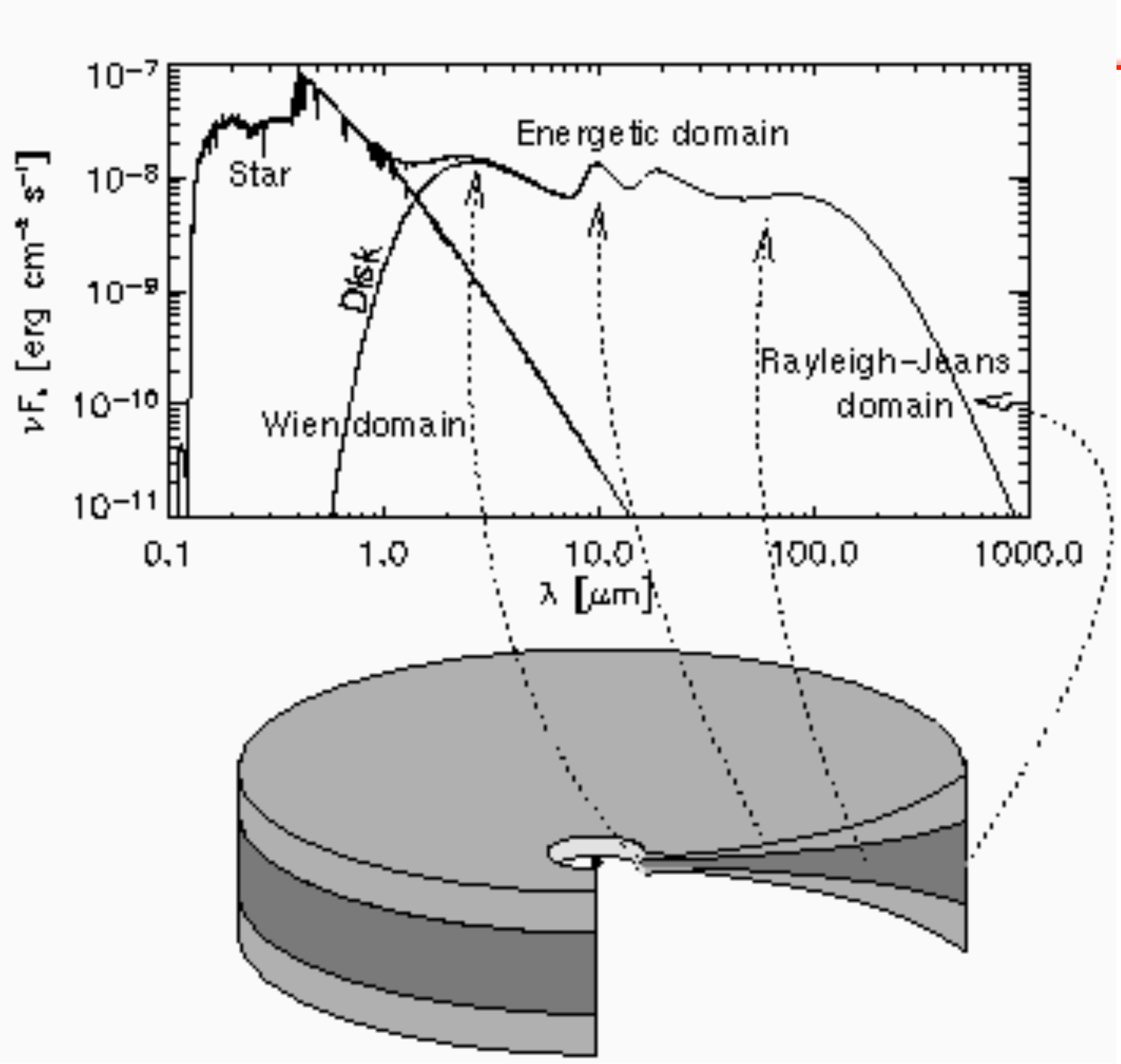
The Goal of Radiative Transfer

Calculate the appearance,

the images, the spectrum,
the polarisation emitted
by a young star and its
circumstellar environment

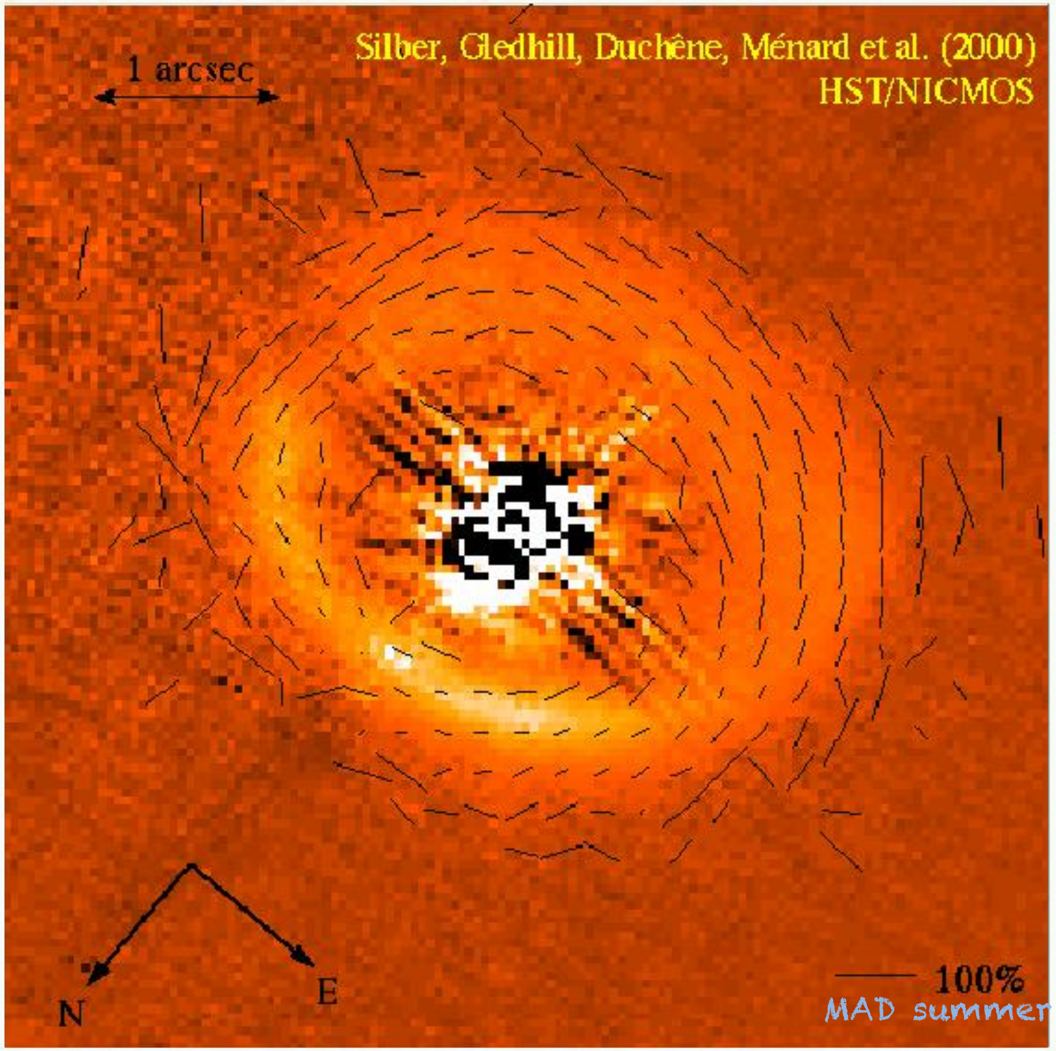


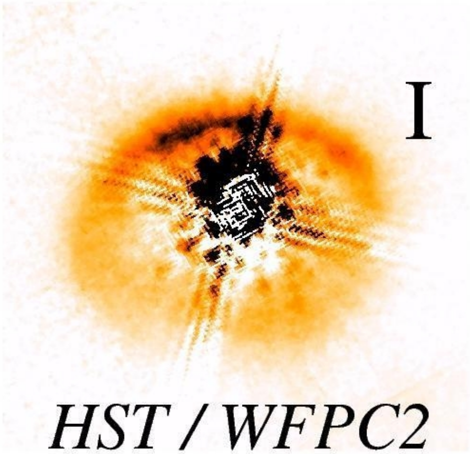
Origin of observed radiation

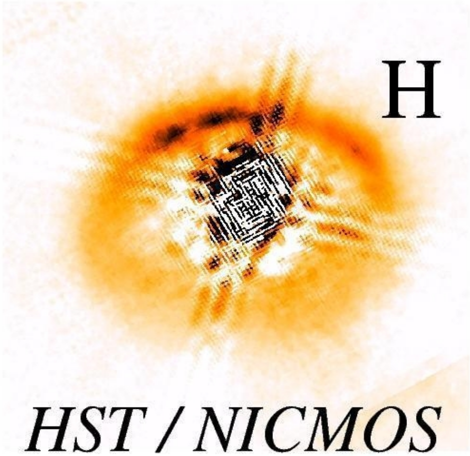


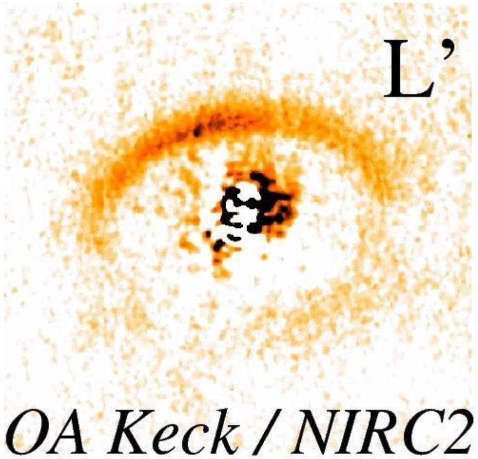
The Goal

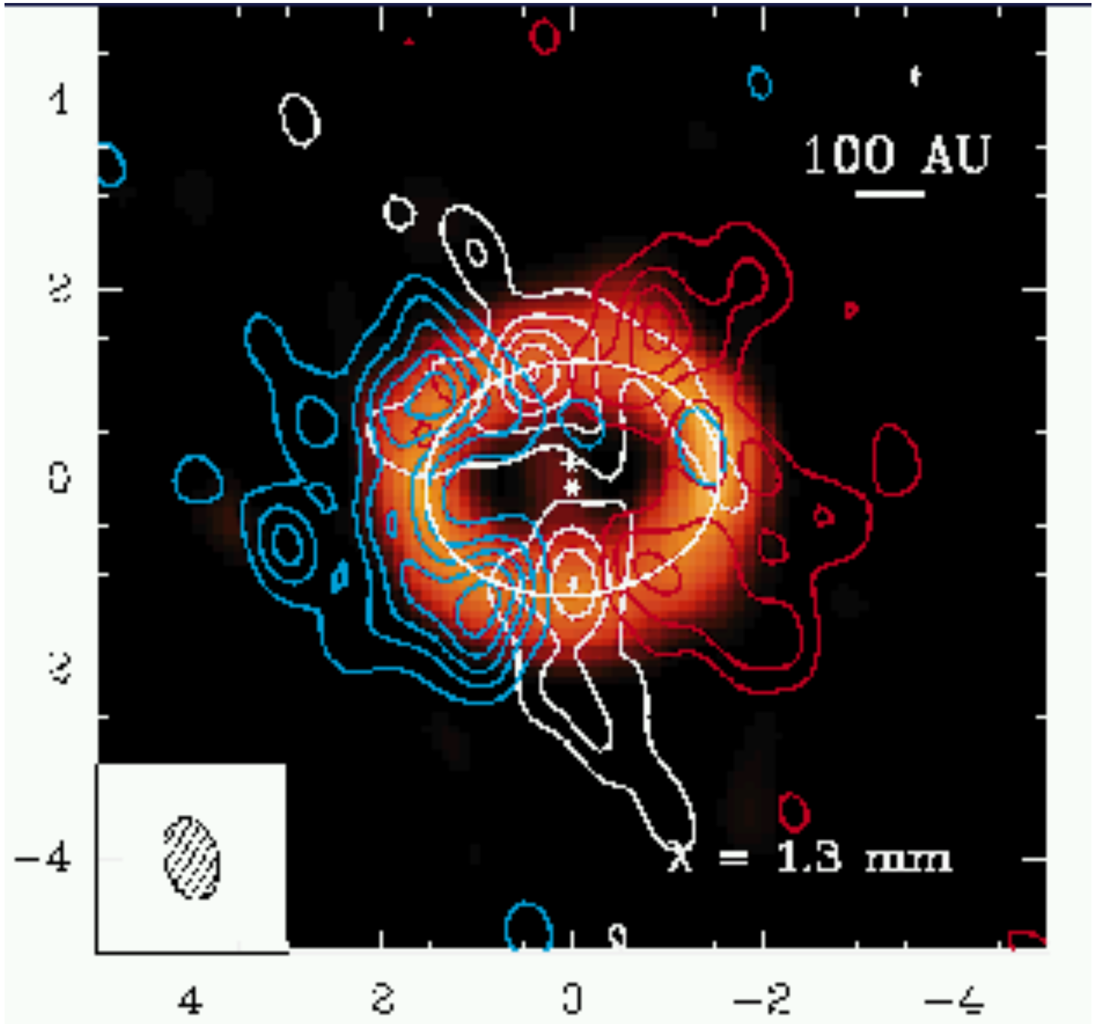
- Model objects like this one: GG Tauri











Ministerio de Economía, Fomento y Turismo

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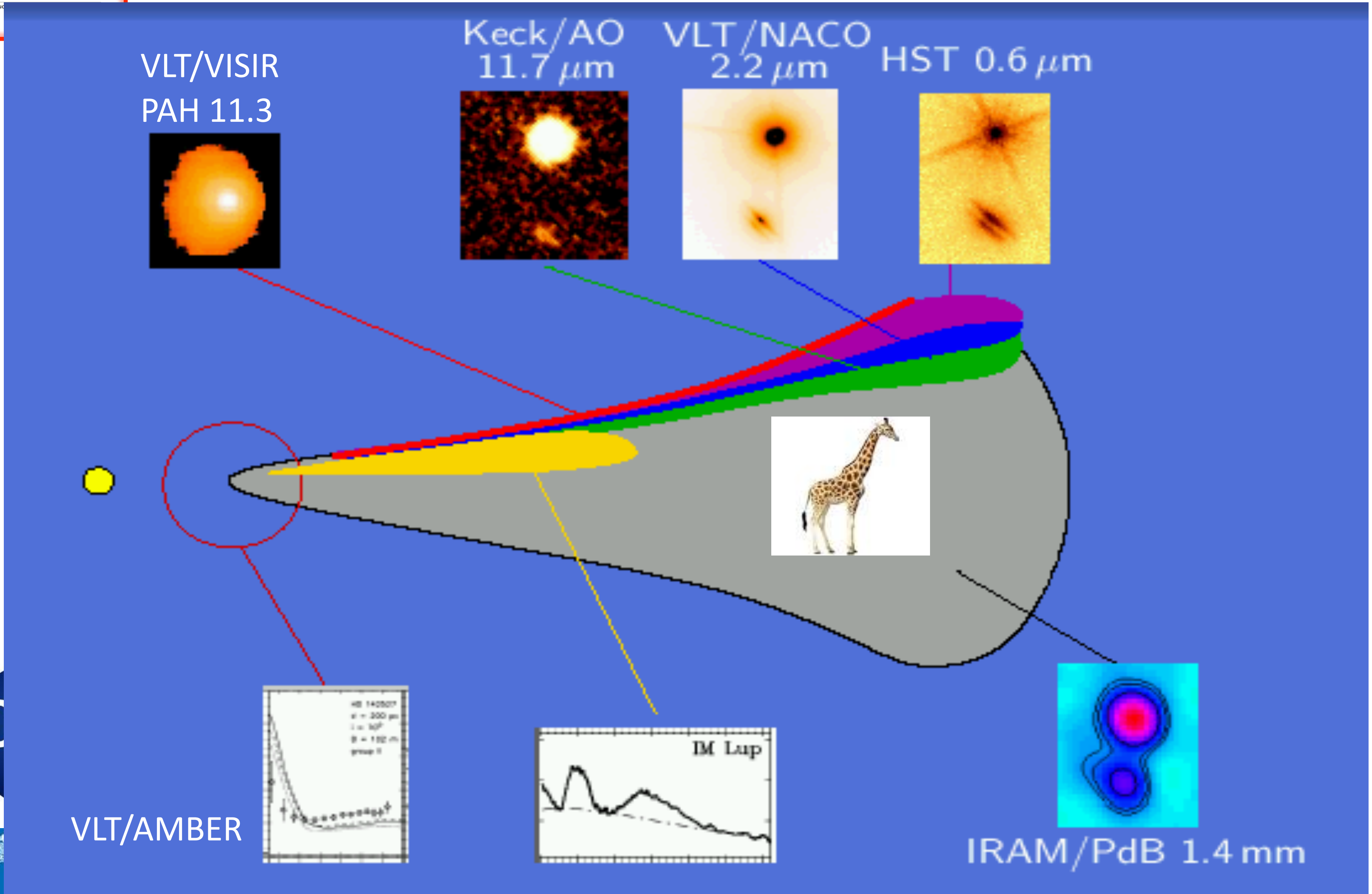
Origin of the radiation



THE GOAL ... in equations

- Calculate the SPECIFIC INTENSITY

$$\mathrm{I}_\lambda(\overrightarrow{r}, \overrightarrow{n})$$

  - at each position,
  - for each direction
  - for each wavelength

The Equation of Radiation Transfer

- To solve the problem , one must solve the following (steady state) equation

$$\frac{\mathrm{d}\mathrm{I}_\lambda(\vec{r},\vec{n})}{\mathrm{d}s} = -\,\kappa_\lambda^{\mathrm{ext}}(\vec{r})\,\mathrm{I}_\lambda(\vec{r},\vec{n}) + \kappa_\lambda^{\mathrm{abs}}(\vec{r})\,\mathrm{B}_\lambda(\mathrm{T}(\vec{r})) + \kappa_\lambda^{\mathrm{diff}}(\vec{r})\,\frac{1}{4\pi}\iint_\Omega \psi_\lambda(\vec{r},\vec{n}',\vec{n})\,\mathrm{I}_\lambda(\vec{r},\vec{n}')\,\mathrm{d}\Omega'$$

$\kappa_\lambda^{\mathrm{ext}}$ Is the EXTINCTION opacity = $\kappa_\lambda^{\mathrm{diff}} + \kappa_\lambda^{\mathrm{abs}}$

$\psi_\lambda(\vec{r},\vec{n}',\vec{n})$ Is the Phase funtion

$\mathrm{B}_\lambda(\mathrm{T}(\vec{r}))$ Is the Planck function

* The formal solution of the ERT is:

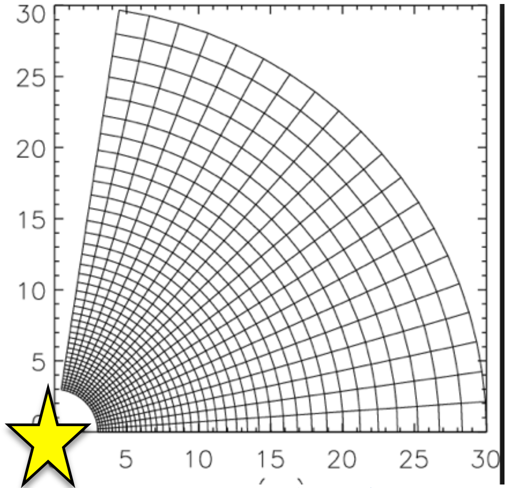
$$\mathrm{I}_\lambda(\overrightarrow{r},\overrightarrow{n}) = \mathrm{I}_\lambda(\overrightarrow{r_0},\overrightarrow{n})\, e^{-\tau_\lambda(\overrightarrow{r_0},\overrightarrow{n},s)} + \int_0^s \varepsilon_\lambda(\overrightarrow{r_0}+s\,\overrightarrow{n},\overrightarrow{n})\, e^{-\left(\tau_\lambda(\overrightarrow{r_0},\overrightarrow{n},s)-\tau_\lambda(\overrightarrow{r_0},\overrightarrow{n},s')\right)}\,\mathrm{d}s'$$

and the emissivity is defined as:

$$\varepsilon_\lambda(\overrightarrow{r},\overrightarrow{n}) = \kappa_\lambda^{\mathrm{abs}}(\overrightarrow{r})\,\mathrm{B}_\lambda(\mathrm{T}(\overrightarrow{r})) + \kappa_\lambda^{\mathrm{diff}}(\overrightarrow{r})\,\frac{1}{4\pi}\iint_\Omega \psi_\lambda(\overrightarrow{r},\overrightarrow{n}',\overrightarrow{n})\,\mathrm{I}_\lambda(\overrightarrow{r},\overrightarrow{n}')\,\mathrm{d}\Omega'$$

- To calculate the thermal emission, one needs to calculate the dust temperature

$$4\pi\, \mathrm{M}_i \int_0^\infty \kappa_i^{\mathrm{abs}}(\lambda) \mathrm{B}_\lambda(\mathrm{T}_i)\, \mathrm{d}\lambda = \Gamma_i^{\mathrm{abs}}$$



- The equations discussed before are sufficient to solve the problem of radiative transfer
  - we can now go ahead and model disks to understand where planets form

- In the next slides I will focus on someting you will not see much in graduate classes.
  - Dust properties , polarisation, and scattered light.
  - these are NECESSARY tools to study disks, because dust particles are the building blocks of planets

Light Scattering, the general case

Stokes formalism

$\boxed{S_{\text{trans}} = \mathbf{M} \times S_{\text{incident}}}$ where **M** is the Mueller matrix

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_t = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{22} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_i$$

All 16 elements (S_{ij}) are independent
+ they are function of wavelength and scattering angle ψ

Not enough observational constraints, simplifications needed

MIE Theory

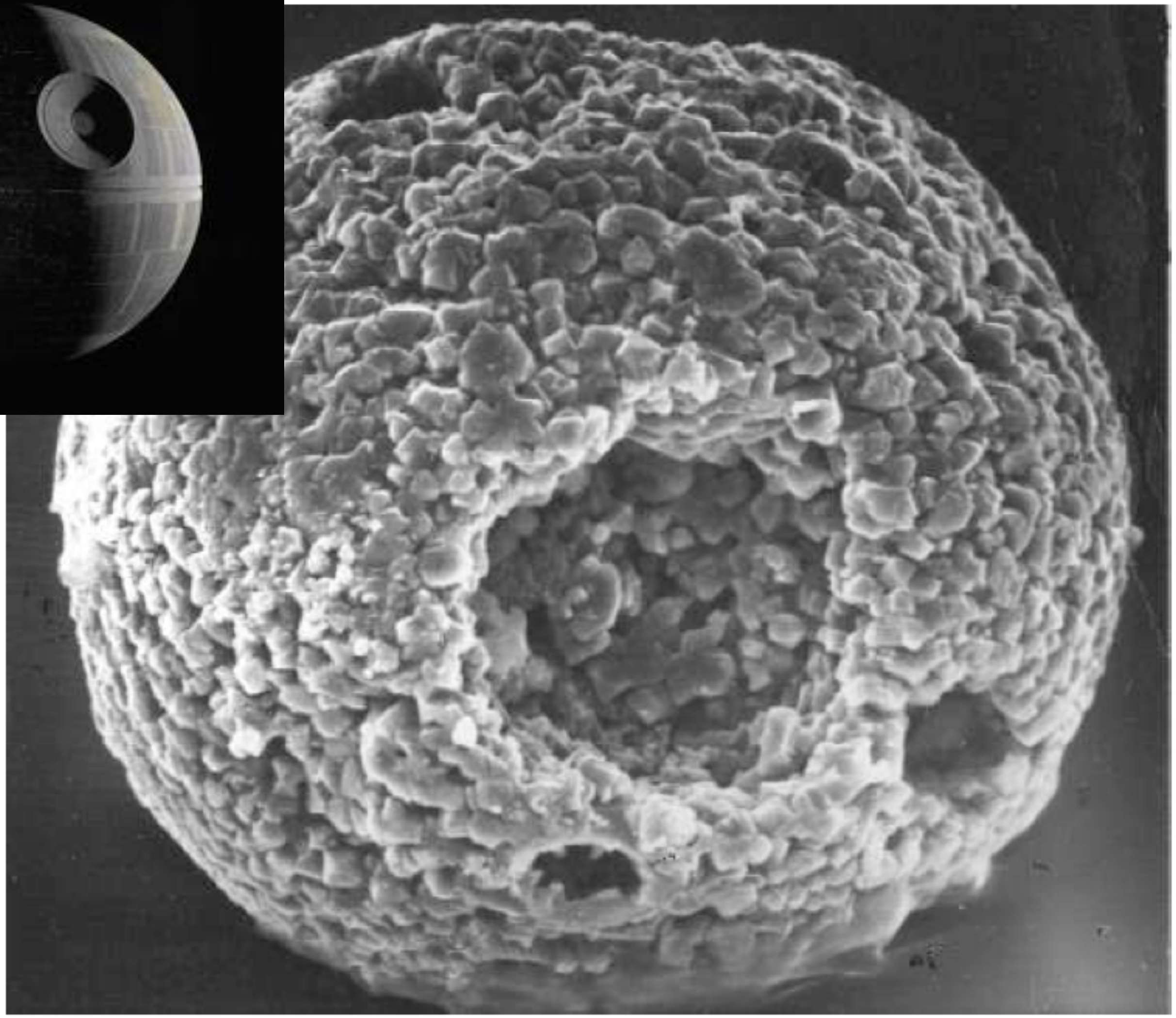
- After Gustav MIE (1908)

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_t = \begin{pmatrix} S_{11} & S_{12} & 0 & 0 \\ S_{12} & S_{11} & 0 & 0 \\ 0 & 0 & S_{33} & S_{34} \\ 0 & 0 & -S_{34} & S_{33} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_i$$

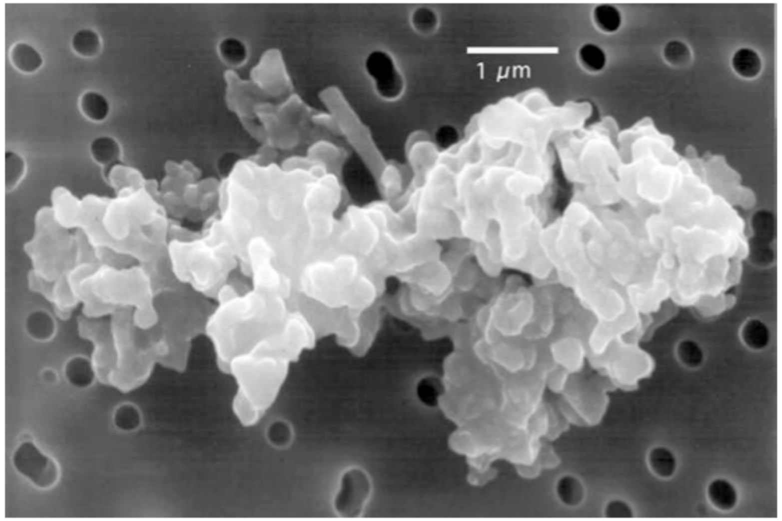
Mie Theory is valid for

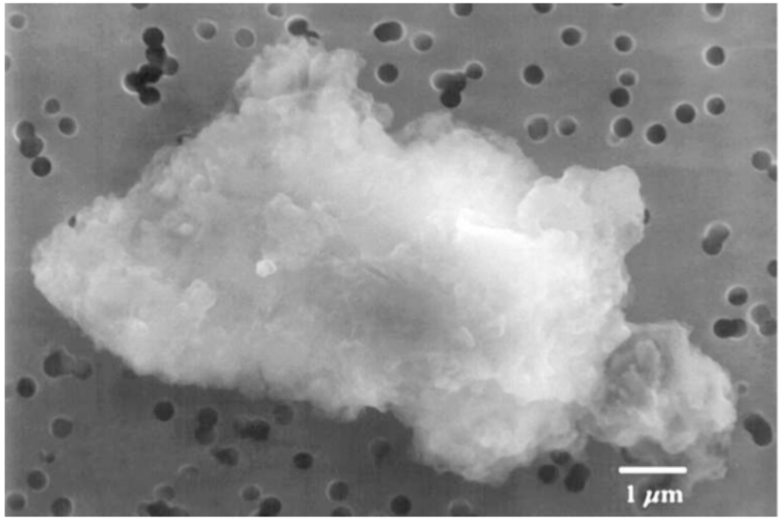
- homogeneous, compact particles
- spherical is shape (or infinite cylinders)

Is MIE Theory a good tool to use ?



Is MIE Theory a good tool to use ?

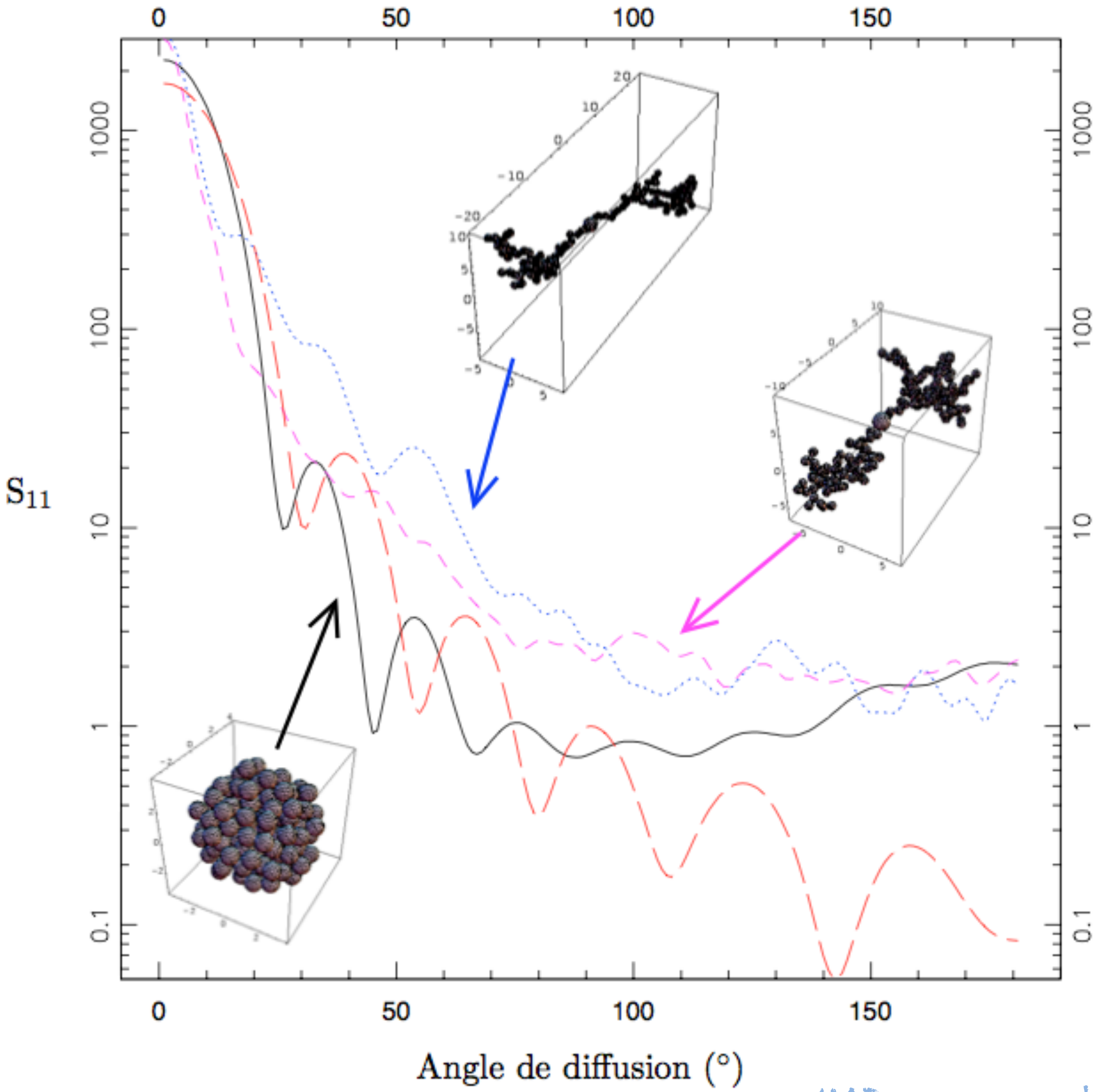




Other Options to MIE Theory

- Discrete Dipole Approximation
  - Very computer intensive
- Generalised Mie Theory
  - Computer intensive
- Distributions of Hollow Spheres
  - A refinement of Mie Theory
- T-Matrix …

Phase functions for aggregates



Implications of Mie Theory

In the case where INCOMING radiation is unpolarised

$$\begin{pmatrix} I \\ Q \\ 0 \\ 0 \end{pmatrix}_t = \begin{pmatrix} S_{11} & S_{12} & 0 & 0 \\ S_{12} & S_{11} & 0 & 0 \\ 0 & 0 & S_{33} & S_{34} \\ 0 & 0 & -S_{34} & S_{33} \end{pmatrix} \begin{pmatrix} I \\ 0 \\ 0 \\ 0 \end{pmatrix}_i$$

Then $I_{trans} = S_{11} I_{inc}$, $Q_{trans} = S_{12} I_{inc}$, $U_{trans} = V_{trans} = 0$

If single scattering:
- -no circular polarisation is produced (V=0)
- -polar usually $\perp$ to scattering plane ($S_{12} > 0$)

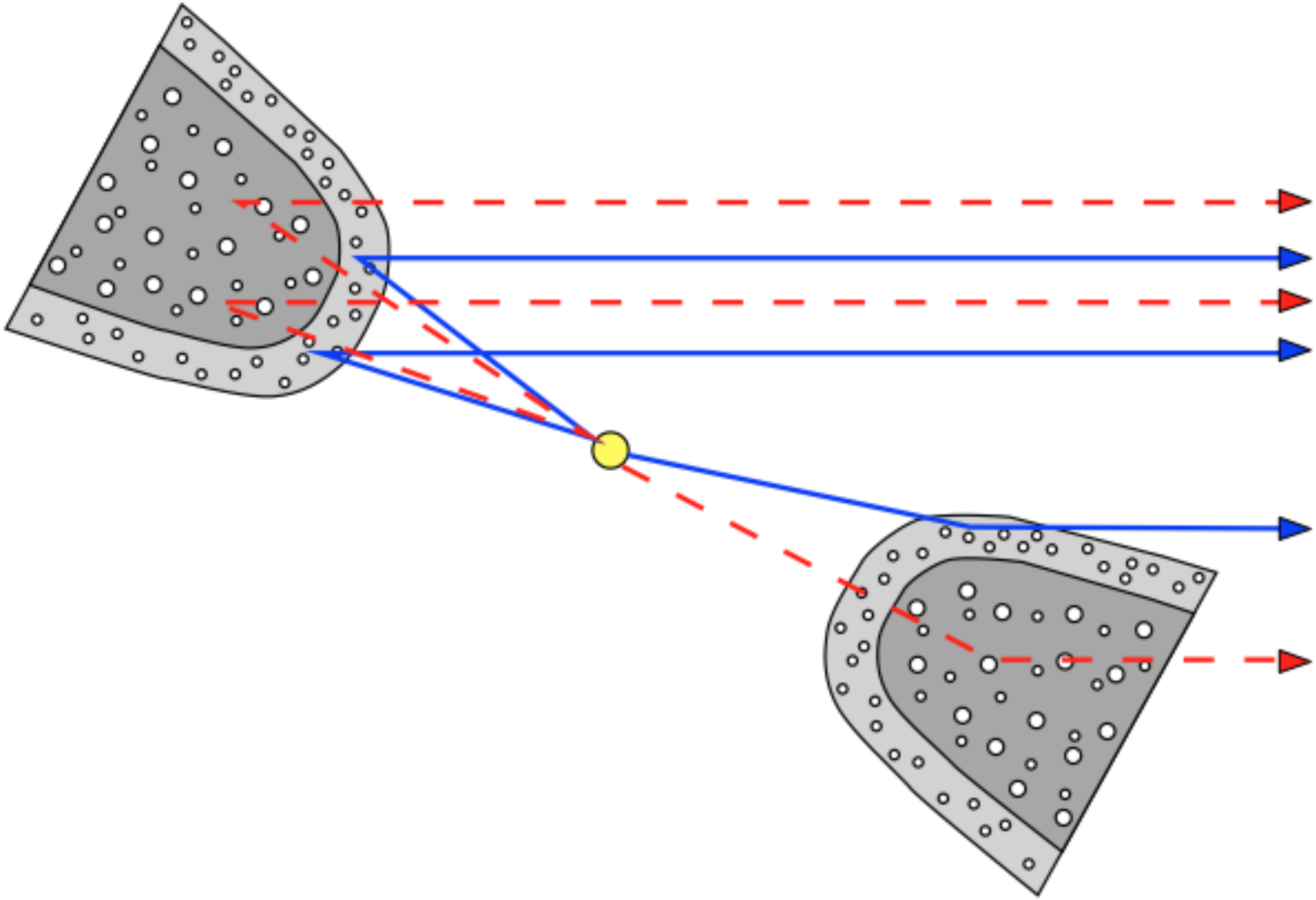
Implications (cont'd)

- In the case where the incident light is POLARISED

$$\begin{pmatrix} S_{11}\, I + S_{12}\, P \\ S_{12}\, I + S_{11}\, P \\ \dots \\ \dots \end{pmatrix}_t = \begin{pmatrix} S_{11} & S_{12} & 0 & 0 \\ S_{12} & S_{11} & 0 & 0 \\ 0 & 0 & S_{33} & S_{34} \\ 0 & 0 & -S_{34} & S_{33} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_i$$

In case of multiple scattering:
- significant photometric errors (up to 30%) can occur
  if Polarisation is neglected!!!
-U et V $\neq$ 0

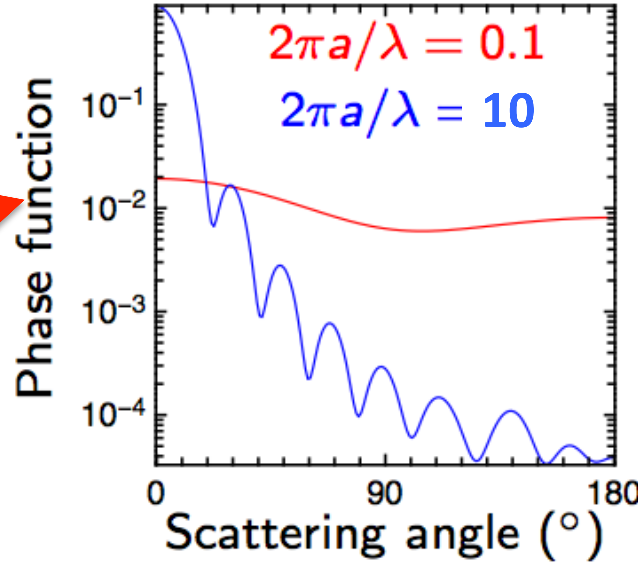
Converting Observations into Disk properties



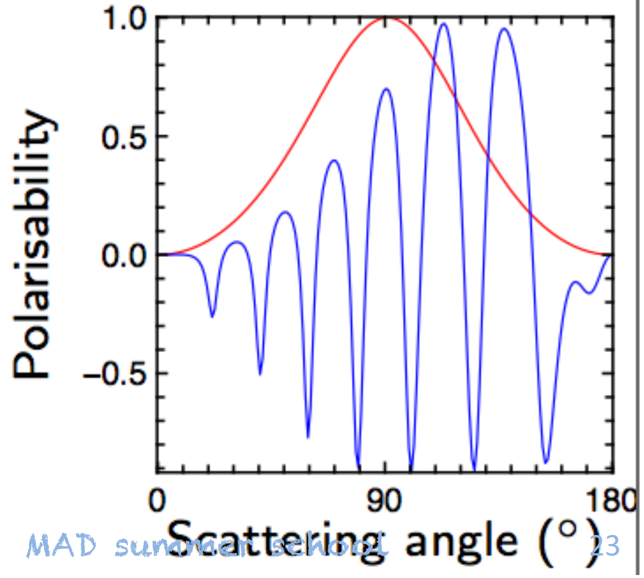
From observations to dust properties

Using the Phase function

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_t = \begin{pmatrix} S_{11} & S_{12} & 0 & 0 \\ S_{12} & S_{11} & 0 & 0 \\ 0 & 0 & S_{33} & S_{34} \\ 0 & 0 & -S_{34} & S_{33} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}_i$$

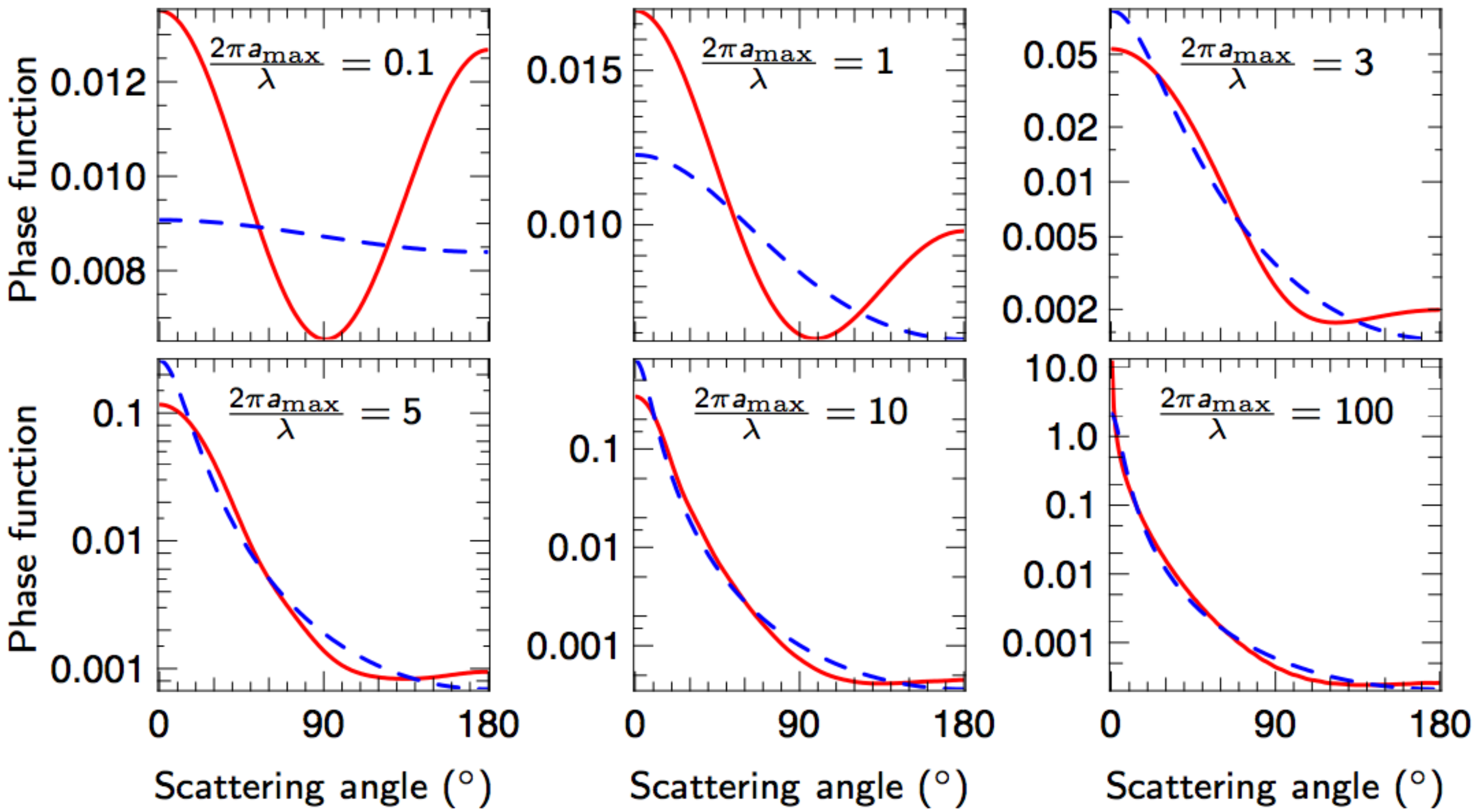


Using the Polarisation

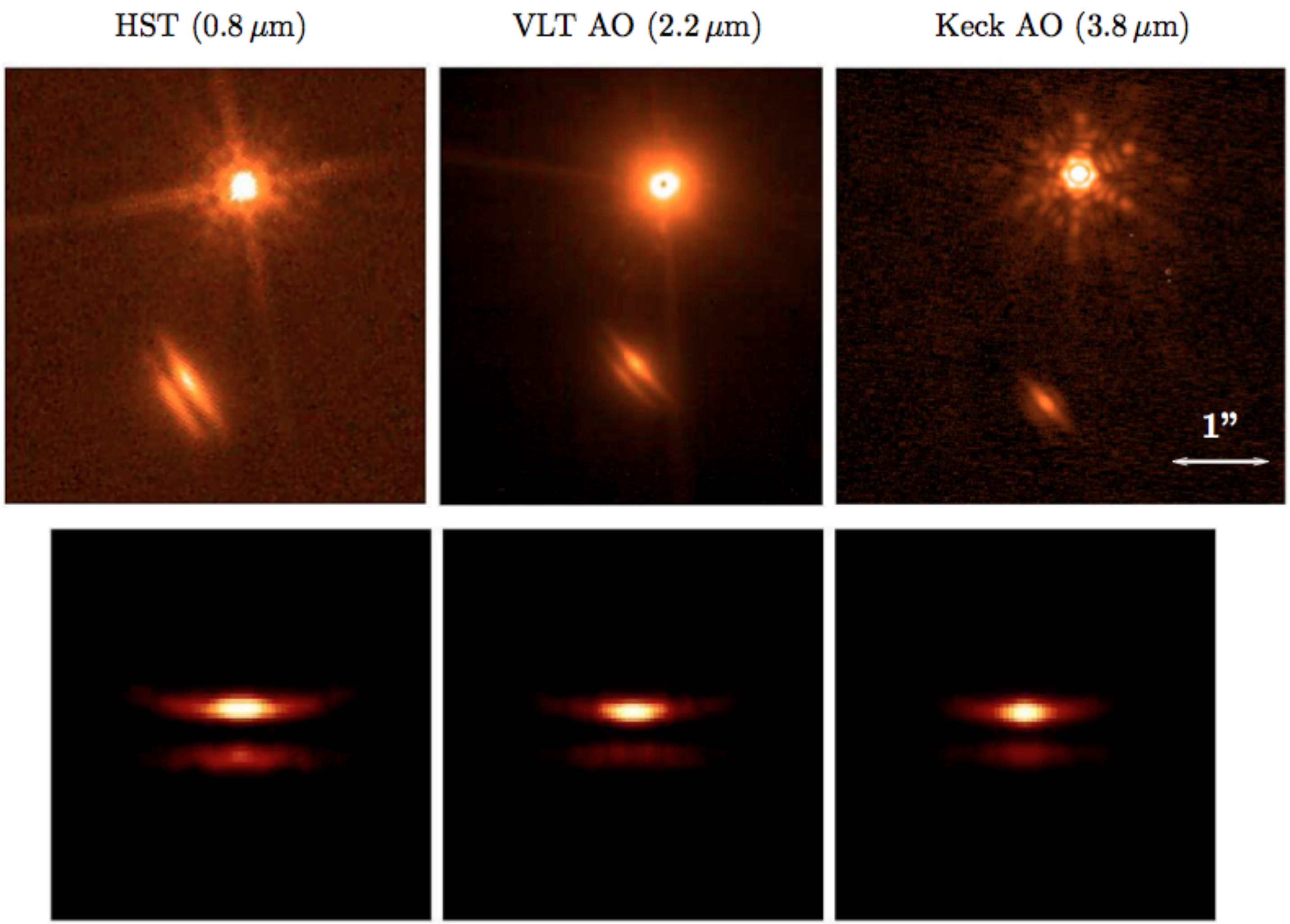


The phase function vs. Dust sizes

Grain size distribution: $dn(a) \propto a^{-3.5}\,da$

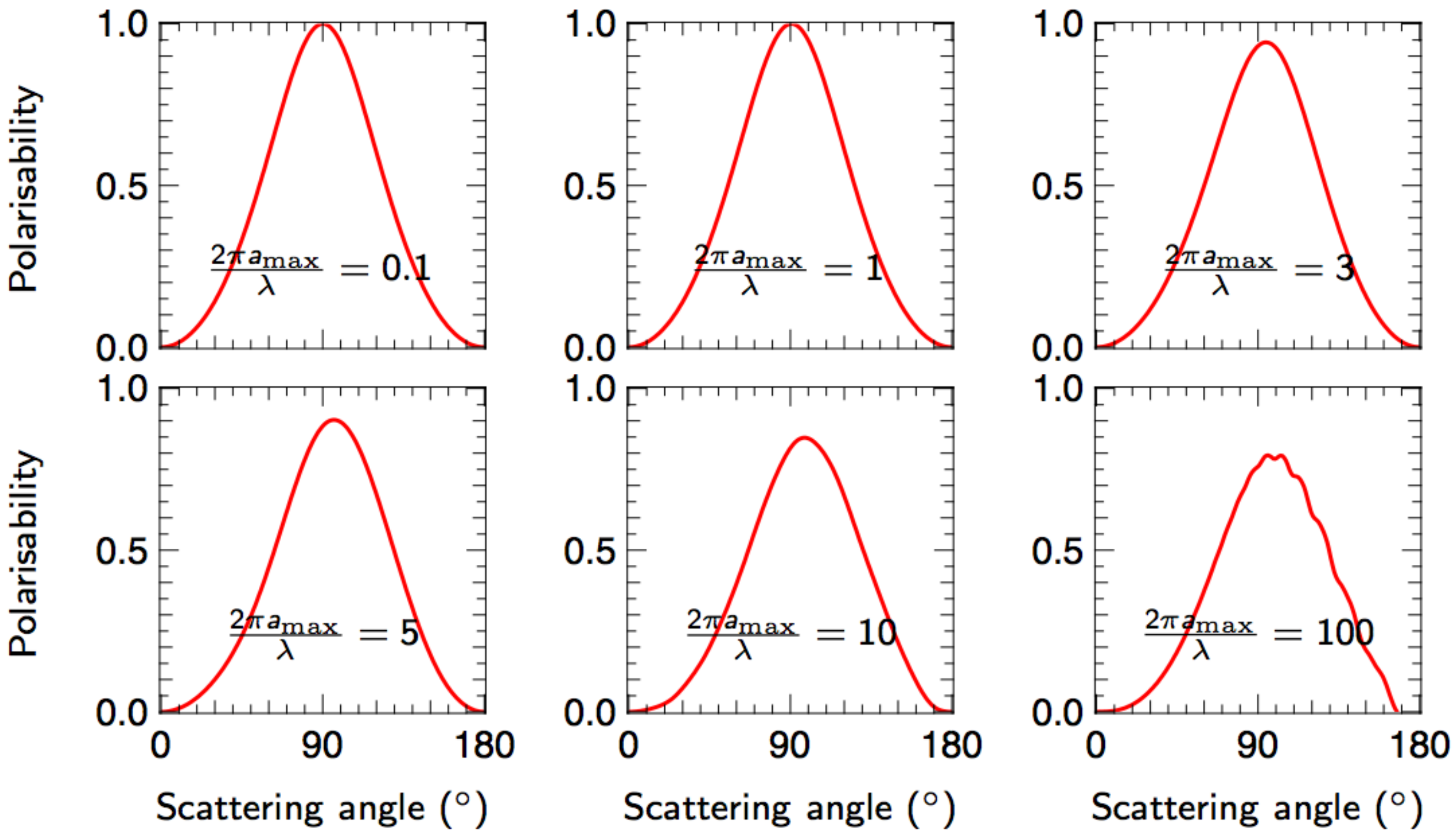


HK Tauri B, an edge-on disk



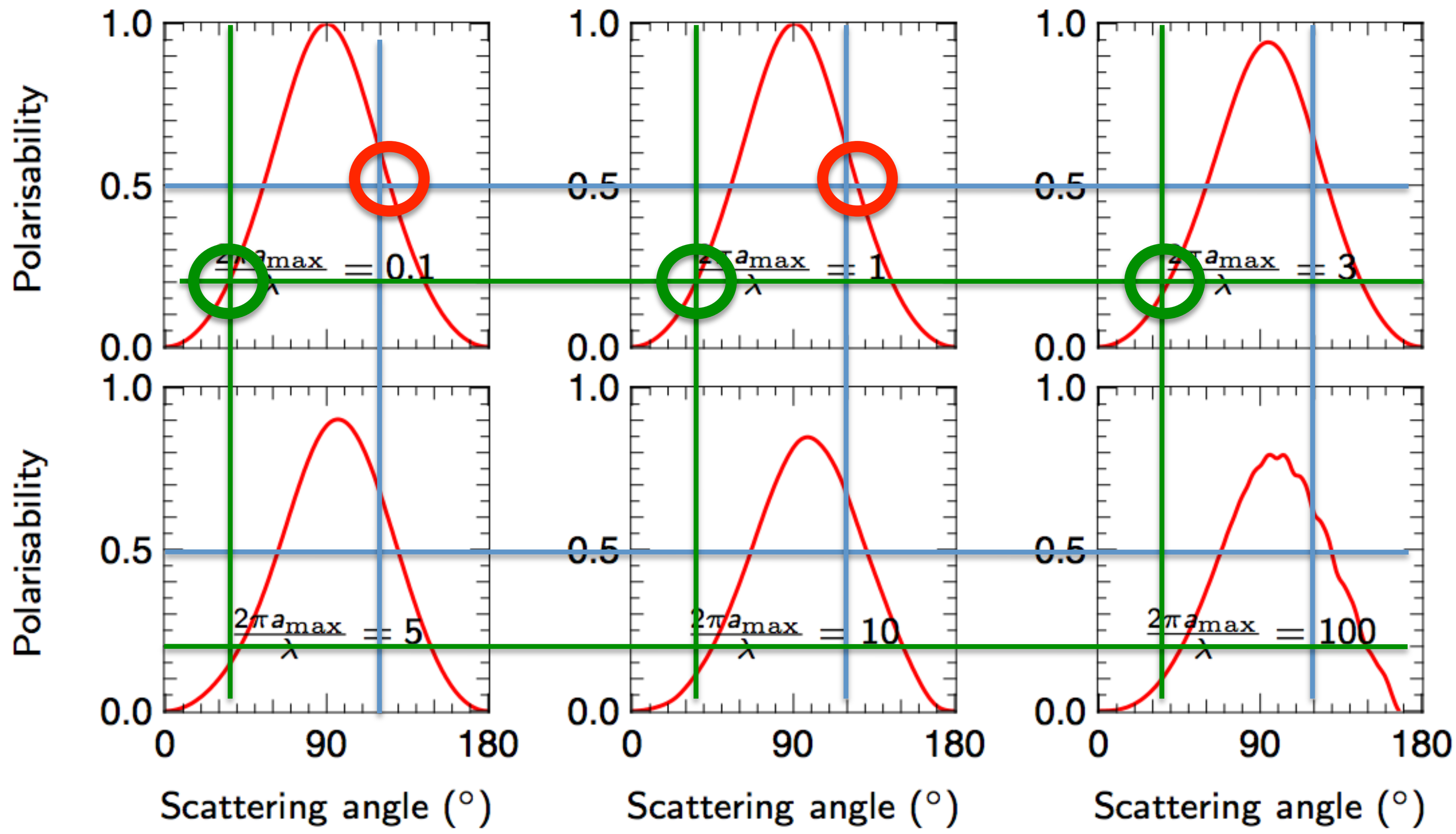
The Polarisation vs. Dust sizes

Grain size distribution: $dn(a) \propto a^{-3.5}\,da$

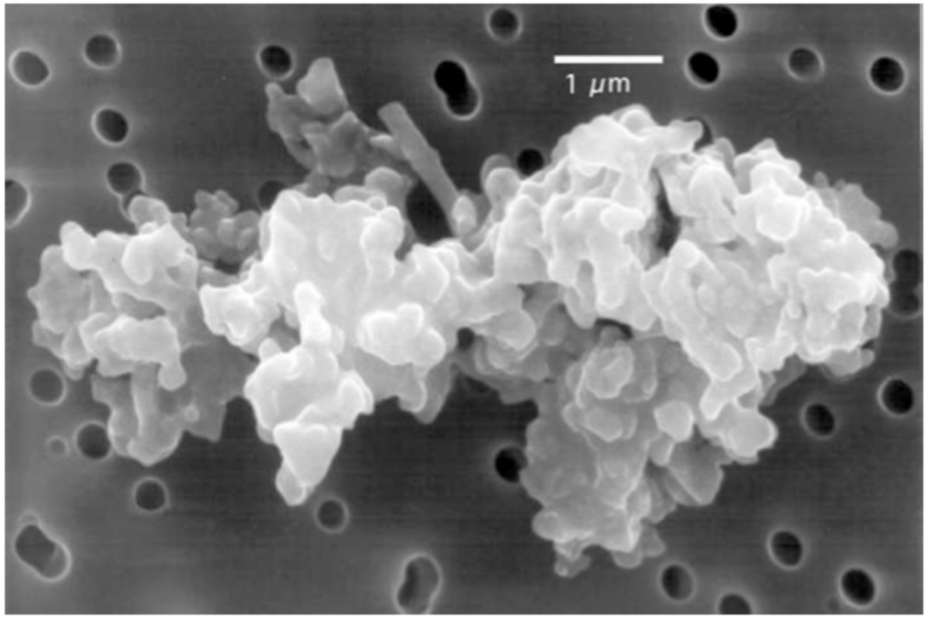


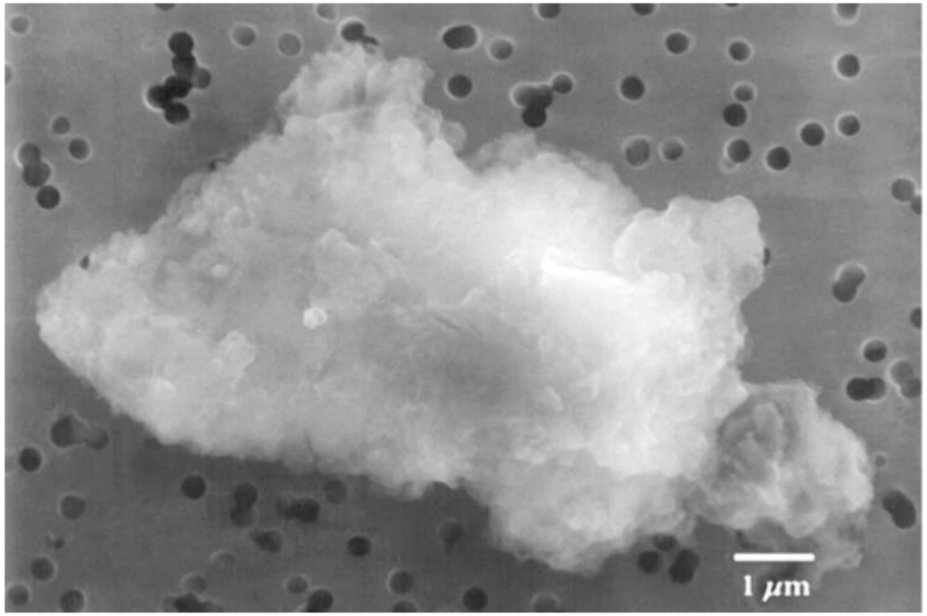
Dust sizes in GG Tau ???

Grain size distribution: $dn(a) \propto a^{-3.5}\,da$



* For comparison, we are looking at particles the size of the small « monomeres » making up IDP aggregates.





Message to remember

- Radiative transfer is needed to understand disks
- I have shown very little, but Good Disk modellers are needed, including in Chile !
- A good code is available , MCFOST and ready to use for new studies
  - Plenty of new developments and new discoveries to be made
  - Requires broad range of competence: Astronomical data, Mineralogy, Electromagnetism, Numerical analysis, Statistics, Computer sciences…

If any of the above is of interest to you, the modelling of disks is for you !